INTEGRATION AS A SUMMATION.

By PROF. GEORGE R. DEAN, Rolla, Mo.

The following method of presenting the matter seems worthy of a place in elementary text-books, but I have not yet seen it in print.

Let the integral $(a, a+h)$ be divided into n equal parts and find the limit, as n is increased without limit, of the expression

$$f(a).\frac{h}{n}+f(a+\frac{h}{n}).\frac{h}{n}+f(a+\frac{2h}{n}).\frac{h}{n}+\ldots+f(a+\frac{(n-1)h}{n})\frac{h}{n}+f(a+h)\frac{h}{n}.$$

Supposing $f(x)$ capable of development by Taylor's Formula, we have

$$f(a+\frac{h}{n})=f(a)+f'(a).\frac{h}{n}+\frac{f''(a)}{2!}\frac{h^2}{n^2}+\ldots+\frac{f^r(a)}{r!}\frac{h^r}{n^r}+\ldots$$

$$f(a+\frac{2h}{n})=f(a)+f'(a)\frac{2h}{n}+\frac{f''(a)2^2h^2}{2!\quad n^2}+\ldots+\frac{f^r(a)2^rh^r}{r!\quad n^r}+\ldots$$

. . . .

$$f(a+\frac{rh}{n})=f(a)+f'(a)\frac{rh}{n}+\frac{f''(a)r^2h^2}{2!\quad n^2}+\ldots+\frac{f^r(a)}{r!}\frac{r^rh^r}{n^r}+\ldots$$

Then $\sum\limits_{r=0}^{r=n} f(a+\frac{rh}{n})\frac{h}{n}=hf(a)+\frac{h^2}{n^2}f'(a)\sum\limits_{r=0}^{r=n} r+\frac{h^3}{n^3}f''(a)\sum\limits_{r=0}^{r=n} r^2+\ldots$

$$+\frac{h^{p+1}}{n^{p+1}}f^p(a)\sum_{r=0}^{r=n} r^p+\ldots$$

By Chrystal's *Algebra*, t. I, p. 487, $\sum\limits_{r=0}^{r=n} r^p=\frac{1}{p+1}n^{p+1}+q_1n^p+\ldots.$

$$\underset{n=\infty}{L}\frac{h^{p+1}}{n^{p+1}}\sum_{r=0}^{r=n} r^p=\frac{h^{p+1}}{p+1}.$$

Then $\underset{n=\infty}{L}\sum\limits_{r=0}^{r=n} f(a+\frac{rh}{n})\frac{h}{n}=hf(a)+\frac{h^2}{2!}f'(a)+\frac{h^3}{3!}f''(a)+\ldots$

If $f(a)$ is the derivative of $\varphi(a)$, we have, by Taylor's Theorem,

$$\varphi(a+h)-\varphi(a)=hf(a)+\frac{h^2}{2!}f'(a)+\frac{h^3}{3!}f''(a)+\ldots$$

$$\therefore\ \varphi(a+h)-\varphi(a)=\underset{n=\infty}{L}\sum_{r=0}^{r=n}f\left(a+\frac{rh}{n}\right)\frac{h}{n}.$$

Hence $\int_a^{a+h}f(x)dx=\varphi(a+h)-\varphi(a)$, where $\varphi(x)$ is the anti-derivative of $f(x)$.

ON THE CHINESE ORIGIN OF THE SYMBOL FOR ZERO.

By PROFESSOR FLORIAN CAJORI.

I have just received a letter from Mr. Y. Mikami, of Tokyo, Japan, containing information which (if confirmed by more extended research) is of great interest and importance. The letter is dated December 15, 1902. From it I quote the following:

"I have found very important relations between the mathematics of India and of China. Arabian numerals seem to be of Chinese origin. The abacus, used by the Chinese from time immemorial, probably afforded the principle of position. In China the use of the symbol 0 for zero seems to have been very old. I desire to study the history of the Chinese mathematics from this point of view, if only I can secure sufficient materials, which is, however, very difficult. Chinese works are not [difficult] to understand for us Japanese, because we use the same letters."

Until recently the symbol for zero and the principle of local value in our notation of numbers were supposed to be of Hindu origin. A few years ago our attention was called to the early work of the Japanese, and now the priority appears to be passing to the Chinese.

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